QP Code: 22/PT/13/IXB(i)

POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-9B(i) : ADVANCED TOPOLOGY

(Pure Mathematics)

(Spl. Paper)

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(Symbols have their usual meanings)

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Prove that a Topological space (X, τ) is compact if every filter in X has a cluster point.

   b) Prove that any Frechet compact T_1-space is countably compact.

   c) Is the family $\{(-n, n)\}_{n \in \mathbb{N}}$ locally finite ? Answer with reasons.

   d) Show that every closed subset of a para-compact space is para-compact.

   e) Is the real number space endowed with lower limit topology metrizable ? Answer with reason.

   f) In an uniform space give an example of a set which is totally bounded but not compact.

   g) Prove that a discrete topological space is para-compact.

2. a) Prove that metrization is invariant under homeomorphism. 5

   b) Let X be a Tychoroff space. It X is locally compact then prove that for every compactification (f, Y) of X, $Y \setminus f(X)$ is closed. 5

3. a) Give an example of a compact Hausdorff space that is not sequentially compact. 4

   b) Prove that a closed subspace of a locally compact space is locally compact. 6

4. a) State and prove Stone-Cech theorem. 8

   b) Give examples to justify that two compactifications of a given topological space may not be homeomorphic. 2

5. a) Show that every Hausdorff para-compact space is regular. 5

   b) Show that for every positive integer n, $\mathbb{R}^n$ with the product topology is metrizable. 5

6. a) Show that every compact subset of a uniform space is totally bounded. 4

   b) Let (X, u) and (Y, q) be uniform spaces and let $f: X \to Y$ be continuous. If X is compact, then show that f is uniformly continuous. 6

7. a) Let (X, τ) be a completely regular space. The show that there exists a proximity δ on X compatible with the topology τ. 6

   b) Show that every open δ-locally finite cover of a topological space has a locally finite refinement. 4